

## New physics in $B \rightarrow \pi\pi$ and $B \rightarrow \pi K$ decays

---

**Seungwon Baek**

*Department of Physics, Yonsei University*

*Seoul 120-749, Korea*

*E-mail: sbaek@cskim.yonsei.ac.kr*

ABSTRACT: We perform a combined analysis of  $B \rightarrow \pi\pi$  and  $B \rightarrow \pi K$  decays with the current experimental data. Assuming SU(3) flavor symmetry and no new physics contributions to the topological amplitudes, we demonstrate that the conventional parametrization in the Standard Model (SM) does not describe the data very well, in contrast with a similar analysis based on the earlier data. It is also shown that the introduction of smaller amplitudes and reasonable SU(3) breaking parameters does not improve the fits much. Interpreting these puzzling behaviors in the SM as a new physics (NP) signal, we study various NP scenarios. We find that when a single NP amplitude dominates, the NP in the electroweak penguin sector is the most favorable. However, other NP solutions, such as NP residing in the QCD-penguin sector and color-suppressed electroweak penguin sector simultaneously, can also solve the puzzle.

KEYWORDS: B-Physics, CP violation, Rare Decays, Beyond Standard Model.

JHEP07(2006)025

---

## Contents

<b>1. Introduction</b>	<b>1</b>
<b>2. SM fitting</b>	<b>2</b>
<b>3. NP fitting</b>	<b>7</b>
<b>4. Conclusions</b>	<b>10</b>

---

## 1. Introduction

In the Standard Model (SM), rare non-leptonic decays  $B \rightarrow \pi\pi$  and  $B \rightarrow \pi K$  provide valuable information on the inner angles of the unitarity triangle of Cabbibo-Kobayashi-Maskawa (CKM) matrix, and have been widely studied. For this purpose the measurement of time-dependent CP-asymmetry given by

$$\frac{\Gamma(\overline{B}(t) \rightarrow f_{\text{CP}}) - \Gamma(B(t) \rightarrow f_{\text{CP}})}{\Gamma(\overline{B}(t) \rightarrow f_{\text{CP}}) + \Gamma(B(t) \rightarrow f_{\text{CP}})} = A_{\text{CP}} \cos(\Delta mt) + S_{\text{CP}} \sin(\Delta mt) \quad (1.1)$$

is essential. Here  $A_{\text{CP}}$  and  $S_{\text{CP}}$  represent direct and indirect CP asymmetries, respectively.

Specifically,  $B \rightarrow \pi\pi$  decays measure the angle  $\alpha$  through the isospin analysis [1]. The information on  $\gamma$  can be obtained from  $B \rightarrow \pi K$  data [2–4]. In addition, if a new physics (NP) beyond the SM exists, it can significantly affect these processes by contributing to penguin amplitudes. Therefore these decay processes are also a sensitive probe of NP [5, 6, 9].

Assuming i) SU(3) flavor symmetry of strong interactions and ii) smallness of annihilation and exchange topologies, Buras, Fleischer, Recksiegel and Schwab [5] concluded the  $B \rightarrow \pi\pi$  and  $B \rightarrow \pi K$  data strongly suggest a NP in the electroweak penguin sector of  $B \rightarrow \pi K$  decay amplitudes. On the other hand, Chiang, Gronau, Rosner and Suprun [10] demonstrated that the  $\chi^2$ -fit to the same data does not show any significant deviation from the SM. It should be noted that Buras et al. [5] assumed that there is no significant NP contribution to  $B \rightarrow \pi\pi$  decays, and used some of  $B \rightarrow \pi\pi$  data with small experimental errors to predict the hadronic parameters of  $B \rightarrow \pi K$  amplitudes. Chiang et al. [10] included all the available  $B \rightarrow \pi\pi$  and  $B \rightarrow \pi K$  data into their fit.

We considered  $B \rightarrow \pi K$  data only [6], and showed that the SM fit faces some difficulties and NP in the electroweak penguin sector is strongly favored in accord with [5]. We concluded that the discrepancy between [5, 6] and [10] is due to the dilution of NP effects by including all the data in [10].

In this paper, we perform  $\chi^2$  fitting to the current data in the SM and also in the presence of NP. We show that even if following the approach of Chiang et al. [10], i.e. the  $\chi^2$  fitting with all the available data, we get much worse  $\chi^2$  fit than in [10]. We calculated  $\Delta\chi^2$  — the contribution of each data point to  $\chi^2$  value — to trace the source of this puzzling behavior. To improve the fit in the SM, we introduce i) smaller amplitudes, and/or ii) reasonable SU(3)-breaking effects to the fits. It turns out that these corrections do not solve the puzzle satisfactorily.

Interpreting these difficulties in the SM fits as a NP signal, we introduce NP parameters, such as, a new weak phase in the amplitudes. We consider various NP scenarios. We introduce three types of NP, NP in the electroweak penguin, color-suppressed electroweak penguin and QCD penguin. When a single NP amplitude dominates, NP in the electroweak penguin is the most favorable solution, supporting the findings in [5]. A given specific NP model, however, contributes to all the NP amplitudes in general. In light of this we also considered the possibility two or more NP amplitudes are enhanced simultaneously.

The paper is organized as follows. In section 2 the SM fittings are considered. In section 3 we perform various NP fittings. The conclusions and discussions are given in section 4.

## 2. SM fitting

The topological amplitudes provide a parametrization for non-leptonic  $B$ -meson decay processes which is independent of theoretical models for the calculation of hadronic matrix elements [11]. The decay amplitudes of  $B \rightarrow \pi\pi$ 's which are  $\bar{b} \rightarrow \bar{d}q\bar{q}$  ( $q = u, d$ ) transitions at quark-level can be written as

$$\begin{aligned}\sqrt{2}A(B^+ \rightarrow \pi^+\pi^0) &= -(T + C + P_{EW} + P_{EW}^C), \\ A(B^0 \rightarrow \pi^+\pi^-) &= -\left(T + P + \frac{2}{3}P_{EW}^C + E + PA\right), \\ \sqrt{2}A(B^0 \rightarrow \pi^0\pi^0) &= -\left(C - P + P_{EW} + \frac{1}{3}P_{EW}^C - E - PA\right).\end{aligned}\tag{2.1}$$

Here  $T, C, P, P_{EW}^{(C)}, E$  and  $PA$  represent tree, color-suppressed tree, QCD-penguin, (color-suppressed) electroweak-penguin, exchange and penguin annihilation diagrams, respectively. Similarly,  $B \rightarrow \pi K$  decays which are  $\bar{b} \rightarrow \bar{s}q\bar{q}$  ( $q = u, d$ ) transitions at quark-level are described by

$$\begin{aligned}A(B^+ \rightarrow \pi^+K^0) &= P' - \frac{1}{3}P'_{EW}{}^C + A', \\ \sqrt{2}A(B^+ \rightarrow \pi^0K^+) &= -\left(P' + T' + C' + P'_{EW} + \frac{2}{3}P'_{EW}{}^C + A'\right), \\ A(B^0 \rightarrow \pi^-K^+) &= -\left(P' + T' + \frac{2}{3}P'_{EW}{}^C\right), \\ \sqrt{2}A(B^0 \rightarrow \pi^0K^0) &= P' - C' - P'_{EW} - \frac{1}{3}P'_{EW}{}^C,\end{aligned}\tag{2.2}$$

where primes indicate  $b \rightarrow s$  transition. The corresponding decay amplitudes for the CP-conjugate modes can be obtained by changing the sign of weak phases while keeping CP-conserving strong phases unchanged.

We can further decompose the QCD penguin diagrams,  $P$  and  $P'$ , depending on the quarks running inside the loop,

$$\begin{aligned}
 P &= V_{ud}V_{ub}^*P_u + V_{cd}V_{cb}^*P_c + V_{td}V_{tb}^*P_t \\
 &= V_{ud}V_{ub}^*(P_u - P_c) + V_{td}V_{tb}^*(P_t - P_c) \\
 &\equiv P_{uc}e^{i\gamma} + P_{tc}e^{-i\beta}, \\
 P' &= V_{us}V_{ub}^*P'_u + V_{cs}V_{cb}^*P'_c + V_{ts}V_{tb}^*P'_t \\
 &= V_{us}V_{ub}^*(P'_u - P'_c) + V_{ts}V_{tb}^*(P'_t - P'_c) \\
 &\equiv P'_{uc}e^{i\gamma} - P'_{tc},
 \end{aligned} \tag{2.3}$$

where we have used the unitarity relation for CKM matrix elements and explicitly written the weak phase dependence for the amplitudes. These notations and conventions will be used throughout the paper. We can estimate the relative sizes of the amplitudes based on the color-, CKM-, and loop-factors,

	$B \rightarrow \pi\pi$	$B \rightarrow \pi K$	
$O(1)$	$ T $	$ P'_{tc} $	
$O(\bar{\lambda})$	$ C ,  P $	$ T' ,  P'_{EW} $	
$O(\bar{\lambda}^2)$	$ P_{EW} $	$ C' ,  P'_{uc} ,  P'_{EW}^C $	(2.4)
$O(\bar{\lambda}^3)$	$ P_{EW}^C $	$ A' $	
$O(\bar{\lambda}^4)$	$ E ,  PA $		

where  $\bar{\lambda}$  is expected to be order of  $0.2 \sim 0.3$ . We will call the decay amplitudes parameterized as in (2.1) and (2.2) and the hierarchy in (2.4) the conventional parametrization in the SM.

The decay amplitudes containing only dominant terms,  $T^{(\prime)}$ ,  $C^{(\prime)}$ ,<sup>1</sup>  $P_{tc}^{(\prime)}$  and  $P'_{EW}$  are given by<sup>2</sup>

$$\begin{aligned}
 \sqrt{2}A(B^+ \rightarrow \pi^+\pi^0) &= -(T + C)e^{i\gamma}, \\
 A(B^0 \rightarrow \pi^+\pi^-) &= -(Te^{i\gamma} + Pe^{-i\beta}), \\
 \sqrt{2}A(B^0 \rightarrow \pi^0\pi^0) &= -C^{i\gamma} + Pe^{-i\beta}, \\
 A(B^+ \rightarrow \pi^+K^0) &= -P', \\
 \sqrt{2}A(B^+ \rightarrow \pi^0K^+) &= P' - T'e^{i\gamma} - C'e^{i\gamma} - P'_{EW}, \\
 A(B^0 \rightarrow \pi^-K^+) &= P' - T'e^{i\gamma}, \\
 \sqrt{2}A(B^0 \rightarrow \pi^0K^0) &= -P' - C'e^{i\gamma} - P'_{EW}.
 \end{aligned} \tag{2.5}$$

Here, we have written  $P_{tc}^{(\prime)}$  as  $P^{(\prime)}$  for the simplicity of notations.

---

<sup>1</sup>We include  $C'$  to the amplitudes, although according to (2.4) it is subdominant. Otherwise, we get extremely poor fit [6]. We hope that this problem will be solved within the SM framework.

<sup>2</sup>We may also think of  $T^{(\prime)}$  and  $C^{(\prime)}$  as  $T^{(\prime)} + P_{uc}^{(\prime)}$  and  $C^{(\prime)} - P_{uc}^{(\prime)}$ , respectively. See [10, 6], for details.

Mode	$BR[10^{-6}]$	$A_{CP}$	$S_{CP}$
$B^+ \rightarrow \pi^+\pi^0$	$5.5 \pm 0.6$	$0.01 \pm 0.06$	
$B^0 \rightarrow \pi^+\pi^-$	$5.0 \pm 0.4$	$0.37 \pm 0.10$	$-0.50 \pm 0.12$
$B^0 \rightarrow \pi^0\pi^0$	$1.45 \pm 0.29$	$0.28 \pm 0.40$	
$B^+ \rightarrow \pi^+K^0$	$24.1 \pm 1.3$	$-0.02 \pm 0.04$	
$B^+ \rightarrow \pi^0K^+$	$12.1 \pm 0.8$	$0.04 \pm 0.04$	
$B^0 \rightarrow \pi^-K^+$	$18.9 \pm 0.7$	$-0.115 \pm 0.018$	
$B^0 \rightarrow \pi^0K^0$	$11.5 \pm 1.0$	$0.02 \pm 0.13$	$0.31 \pm 0.26$

**Table 1:** The current experimental data for CP averaged branching ratios ( $BR$ ), direct CP asymmetries ( $A_{CP}$ ) and indirect CP asymmetries ( $S_{CP}$ ) for  $B \rightarrow \pi\pi$  and  $B \rightarrow \pi K$  decays filed by HFAG [14].

The current experimental data for the CP-averaged branching ratio ( $BR$ ), the direct CP-asymmetry ( $A_{CP}$ ) and the indirect CP-asymmetry ( $S_{CP}$ ) are shown in table 1. It immediately shows some puzzling behaviors which are difficult to understand if we believe (2.4) and (2.5). Firstly, (2.4) suggests that  $BR(B^0 \rightarrow \pi^0\pi^0)$  should be about 3 times lower than the data ( $B \rightarrow \pi\pi$  puzzle) [9, 15]. Secondly, the ratios

$$\begin{aligned}
 R_c &\equiv \frac{2BR(B^+ \rightarrow \pi^0K^+)}{BR(B^+ \rightarrow \pi^+K^0)}, \\
 R_n &\equiv \frac{BR(B^0 \rightarrow \pi^-K^+)}{2BR(B^0 \rightarrow \pi^0K^0)}
 \end{aligned}
 \tag{2.6}$$

should equal to a good approximation. However, the data shows about  $1.5\sigma$  difference. We should say that this so-called  $R_c/R_n$  problem is not so statistically significant now. Thirdly, we expect from (2.5) that

$$A_{CP}(B^+ \rightarrow \pi^0K^+) \approx A_{CP}(B^0 \rightarrow \pi^-K^+).
 \tag{2.7}$$

The data deviate from this relation by about  $2.7\sigma$  level. Finally, the dominant terms in  $S_{CP}(B^0 \rightarrow \pi^0K^0)$  gives  $\sin 2\beta$  which is quite precisely measured from  $b \rightarrow s\bar{c}$  modes to be  $\sin 2\beta = 0.685 \pm 0.032$  [14]. The current data shows about  $1.43\sigma$  difference. These last three are usually called “ $B \rightarrow \pi K$  puzzle” [16].

Assuming the exact SU(3) flavor symmetry, we can relate the topological amplitudes of  $B \rightarrow \pi\pi$  decays to the corresponding amplitudes of  $B \rightarrow \pi K$  decays as follows:

$$\begin{aligned}
 \frac{T}{T'} &= \frac{C}{C'} = \frac{V_{ud}}{V_{us}}, \\
 \frac{P}{P'} &= \left| \frac{V_{td}}{V_{ts}} \right|.
 \end{aligned}
 \tag{2.8}$$

In addition, it is known that the Wilson coefficients for the electroweak penguins  $c_7$  and  $c_8$  are much smaller than  $c_9$  and  $c_{10}$  [12] in the SM, which leads to a relation between the electroweak penguin diagrams and trees in the SU(3)-limit [13],

$$P'_{EW} = \frac{3}{4} \frac{c_9 + c_{10}}{c_1 + c_2} R(T' + C') + \frac{3}{4} \frac{c_9 - c_{10}}{c_1 - c_2} R(T' - C'),$$

	SM fit I	SM fit II	SM fit III
$\chi^2_{\min}/dof$ (quality of fit)	18.8/10 (4.3%)	0.62/5 (99%)	16.4/8 (3.7%)
$\gamma$	$69.4^\circ \pm 5.8^\circ$	$73.2^\circ \pm 5.2^\circ$	$70.6^\circ \pm 5.2^\circ$
$ T' $ (eV)	$5.22 \pm 0.26$	$5.26 \pm 0.27$	$6.59 \pm 0.29$
$\delta_{T'}$	$28.3^\circ \pm 4.8^\circ$	$29.9^\circ \pm 5.3^\circ$	$25.0^\circ \pm 6.8^\circ$
$ C' $ (eV)	$3.82 \pm 0.48$	$3.20 \pm 0.52$	$4.02 \pm 0.44$
$\delta_{C'}$	$-40.2^\circ \pm 9.7^\circ$	$-14.3^\circ \pm 18.0^\circ$	$-47.0^\circ \pm 10.5^\circ$
$ P' $ (eV)	$48.9 \pm 0.7$	$47.2 \pm 1.7$	$36.9 \pm 4.8$
$ P'_{uc} $ (eV)	-	-	$24.1 \pm 6.4$
$\delta_{P'_{uc}}$ (eV)	-	-	$178^\circ \pm 2^\circ$

**Table 2:** Results for “SM fit I”, “SM fit II” and “SM fit III”. See the text for details.

$$P'_{EW} = \frac{3}{4} \frac{c_9 + c_{10}}{c_1 + c_2} R(T' + C') - \frac{3}{4} \frac{c_9 - c_{10}}{c_1 - c_2} R(T' - C'). \quad (2.9)$$

Here,  $R$  is given by a combination of CKM matrix elements,

$$R = \left| \frac{V_{ts} V_{tb}^*}{V_{us} V_{ub}^*} \right| = \frac{1}{\lambda^2} \frac{\sin(\beta + \gamma)}{\sin \beta}. \quad (2.10)$$

Using these SU(3) relations, we have 6 parameters to fit in (2.5):  $|T'|$ ,  $|C'|$ ,  $|P'|$ , two relative strong phases and  $\gamma$  (**SM fit I**). Now we can perform the fit to the current experimental data which are given in table 1. Since in  $P'_{EW}$  is neglected in (2.5), we use for this fit

$$P'_{EW} = \frac{3}{2} \frac{c_9 + c_{10}}{c_1 + c_2} R(T' + C'), \quad P'_{EW} = 0. \quad (2.11)$$

The inner angle  $\beta$  of the unitarity triangle is strongly constrained and is given by  $\sin 2\beta = 0.725 \pm 0.018$ , so we fixed  $\beta = 23.22^\circ$ . Other parameters used as inputs for the fit are as follows:  $\lambda = 0.226$ ,  $c_1 = 1.081$ ,  $c_2 = -0.190$ ,  $c_9 = -1.276\alpha_{em}$ ,  $c_{10} = 0.288\alpha_{em}$ . The result for this fit is shown in table 2.<sup>3</sup> It can be seen that the SM with exact SU(3) symmetry gives  $\chi^2_{\min}/dof = 18.8/10(4.3\%)$ , which is quite a poor fit. To trace the observables which make the fit poor, we list  $\Delta\chi^2_{\min}$  — the contribution of each data point to the  $\chi^2_{\min}$  in table 3. From table 3 we can see that the observables,  $BR(B^0 \rightarrow \pi^0 K^0)$ ,  $A_{CP}(B^+ \rightarrow \pi^0 K^+)$  and  $S_{CP}(B^0 \rightarrow \pi^0 K^0)$  which caused the  $B \rightarrow \pi K$  puzzles are exactly those with large  $\Delta\chi^2_{\min}$ . They are about  $1.8\sigma \sim 2.2\sigma$  away from the best fit values.

An alternative way to see the discrepancy between the SM and the experiments is to remove the observables which give large  $\Delta\chi^2_{\min}$  from the fit and predict them from the fitted parameters of the remaining observables. For example, we dropped the data for  $BR(B^+ \rightarrow \pi^+ K^0)$ ,  $BR(B^0 \rightarrow \pi^- K^+)$ ,  $BR(B^0 \rightarrow \pi^0 K^0)$ ,  $A_{CP}(B^+ \rightarrow \pi^0 K^+)$  and  $S_{CP}(B^0 \rightarrow \pi^0 K^0)$  whose  $\Delta\chi^2_{\min}$ 's from “SM fit I” are greater than 1 from the  $\chi^2$  fitting.

<sup>3</sup>We have also checked the case where  $P'_{EW} \approx -\frac{3}{2} \frac{(c_9 + c_{10})}{(c_1 + c_2)} q_{EW} R T'$  and  $q_{EW}$  is fitted as in [10]. In this case we obtained  $q_{EW} = 0.36 \pm 0.33$  which is far away from the SM expectation  $\delta_{EW} = 1$ . Therefore we used the exact SU(3) relation (2.9) for this fit.

Observable	SM fit I	SM fit II	SM fit III
$BR(B^+ \rightarrow \pi^+\pi^0)$	0.67	0.00074	0.24
$BR(B^0 \rightarrow \pi^+\pi^-)$	0.31	0.0086	0.068
$BR(B^0 \rightarrow \pi^0\pi^0)$	0.83	0.0013	0.46
$BR(B^+ \rightarrow \pi^+K^0)$	1.2	-	0.17
$BR(B^+ \rightarrow \pi^0K^+)$	0.025	0.00679	0.59
$BR(B^0 \rightarrow \pi^-K^+)$	1.3	-	0.98
$BR(B^0 \rightarrow \pi^0K^0)$	4.8	-	1.4
$A_{CP}(B^+ \rightarrow \pi^+\pi^0)$	0.028	0.028	0.028
$A_{CP}(B^0 \rightarrow \pi^+\pi^-)$	$9.9 \times 10^{-5}$	0.10	1.1
$A_{CP}(B^0 \rightarrow \pi^0\pi^0)$	0.50	0.029	0.40
$A_{CP}(B^+ \rightarrow \pi^+K^0)$	0.25	0.25	0.18
$A_{CP}(B^+ \rightarrow \pi^0K^+)$	3.1	-	3.1
$A_{CP}(B^0 \rightarrow \pi^-K^+)$	0.68	0.044	1.2
$A_{CP}(B^0 \rightarrow \pi^0K^0)$	0.33	0.15	0.49
$S_{CP}(B^0 \rightarrow \pi^+\pi^-)$	0.85	0.00013	0.19
$S_{CP}(B^0 \rightarrow \pi^0K^0)$	3.9	-	5.9

**Table 3:**  $\Delta\chi_{\min}^2$  — the contribution of each data point to the  $\chi_{\min}^2$ .

The results for this approach (**SM fit II**) are shown in the 2nd columns of tables 2 and 3. From table 2, we can see the quality of fitting has improved dramatically while the values of parameters are consistent with those of “SM fit I”. Also table 3 shows that all the observables considered are excellently described by the SM parametrization. Now we can predict the omitted observables from the fitted values in table 2. The predictions (deviation from the best fit values) for  $BR(B^+ \rightarrow \pi^+K^0)$ ,  $BR(B^0 \rightarrow \pi^-K^+)$ ,  $BR(B^0 \rightarrow \pi^0K^0)$ ,  $A_{CP}(B^+ \rightarrow \pi^0K^+)$  and  $S_{CP}(B^0 \rightarrow \pi^0K^0)$  are  $21.0 \pm 0.6$  ( $2.1\sigma$ ),  $18.5 \pm 0.4$  ( $0.46\sigma$ ),  $8.17 \pm 0.16$  ( $3.3\sigma$ ),  $-0.065 \pm 0.002$  ( $2.6\sigma$ ) and  $0.81 \pm 0.0001$  ( $1.9\sigma$ ), respectively. These deviations imply that the  $B \rightarrow \pi K$  puzzles are more serious than the estimations given below (2.6) and below (2.7).

Until now we have assumed exact SU(3) flavor symmetry. Before considering NP as a solution of these  $B \rightarrow \pi K$  puzzles, we proceed to improve the SM parametrization by including smaller amplitudes we have neglected in the above analysis and/or by taking SU(3) breaking effects into account. First we include  $P'_{uc}$  and  $P'_{EW}{}^C$  which are subdominant according to (2.4) in the decay amplitudes. Then we the decay amplitudes of  $B \rightarrow \pi\pi$  and  $B \rightarrow \pi K$  are corrected to be

$$\begin{aligned}
 \sqrt{2}A(B^+ \rightarrow \pi^+\pi^0) &= -(T + C)e^{i\gamma}, \\
 A(B^0 \rightarrow \pi^+\pi^-) &= -(Te^{i\gamma} + Pe^{-i\beta}), \\
 \sqrt{2}A(B^0 \rightarrow \pi^0\pi^0) &= -C^{i\gamma} + Pe^{-i\beta}, \\
 A(B^+ \rightarrow \pi^+K^0) &= -P' - \frac{1}{3}P'_{EW}{}^C + P'_{uc}e^{i\gamma},
 \end{aligned}$$

$$\begin{aligned}
\sqrt{2}A(B^+ \rightarrow \pi^0 K^+) &= P' - P'_{EW} - \frac{2}{3}P'_{EW}{}^C - (T' + C' + P'_{uc}) e^{i\gamma}, \\
A(B^0 \rightarrow \pi^- K^+) &= P' - \frac{2}{3}P'_{EW}{}^C - (T' + P'_{uc}) e^{i\gamma}, \\
\sqrt{2}A(B^0 \rightarrow \pi^0 K^0) &= -P' - P'_{EW} - \frac{1}{3}P'_{EW}{}^C - (C' - P'_{uc}) e^{i\gamma}.
\end{aligned}
\tag{2.12}$$

We also incorporate the factorizable SU(3) breaking effect to the tree amplitude [10] so that

$$\frac{T}{T'} = \frac{f_\pi}{f_K} \frac{V_{ud}}{V_{us}},
\tag{2.13}$$

where  $f_\pi$  ( $f_K$ ) is the decay constant of  $\pi$  ( $K$ ). For numerical analysis we used  $f_{\pi(K)} = 131(160)$  (MeV). For color-suppressed tree and QCD penguin amplitudes we still use the SU(3) relation (2.8). We also use the relation (2.9) for electroweak penguins in terms of trees (**SM fit III**).

As can be seen in table 2, the  $\chi_{\min}^2$  does not improve at all. In addition  $|P'_{uc}|$  which should be much smaller compared with  $|T'|$  does not follow this hierarchy. We also see that  $\Delta\chi_{\min}^2$ 's for  $S_{CP}(B^0 \rightarrow \pi^0 K^0)$  and  $A_{CP}(B^+ \rightarrow \pi^0 K^+)$  in table 3 are still troublesome. Therefore we conclude that the inclusion of factorizable SU(3) breaking effect in  $T^{(\prime)}$  and smaller amplitudes  $P'_{uc}$  and  $P'_{EW}{}^C$  alone does not help improving the SM fit.

Now we consider the effect of reasonable SU(3) breaking. To do this we introduce two parameters  $b_C$  and  $b_P$  to represent the SU(3) breaking for the color-suppressed tree and QCD penguin so that

$$\frac{C}{C'} = b_C \frac{V_{ud}}{V_{us}}, \quad \frac{P}{P'} = b_P \left| \frac{V_{td}}{V_{ts}} \right|.
\tag{2.14}$$

We added two free parameters  $b_C$  and  $b_P$  to ‘‘SM fit III’’ (**SM fit IV**) and obtained  $b_C = 3.5 \pm 6.6$  and  $b_P = 1.7 \pm 0.7$  ( $\chi_{\min}^2/dof = 4.7/5$ ). Although they have huge errors, the central values require too large SU(3) breaking effect, considering the fact that it is expected to be at most 20 – 30%. To make matters worse, not only  $|P'_{uc}|$  is too large but  $\gamma = 41^\circ \pm 5^\circ$  is much lower than that obtained in the global CKM fitting [17].

### 3. NP fitting

We have seen in section 2 that the SM parametrization does not describe the experimental data very well. Although the discrepancy is about 2 – 3  $\sigma$  level and we cannot rule out the SM yet, it would be interesting to investigate whether a new parametrization coming from NP will improve the fitting.

Since the parameterizations in (2.5) and (2.12) can perfectly fit to the  $B \rightarrow \pi\pi$  data, we will assume that NP appears only in the  $B \rightarrow \pi K$  modes. A given NP model can generate many new terms in the decay amplitudes with their own weak phases and strong phases. To simplify the analysis we adopt a reasonable argument that the strong phases of NP are negligible [6, 7]. However, it should be noted that this argument is not rigorous enough to allow the estimation of strong phases and also has a caveat to keep in mind [8].



	NP fit I	NP fit II	NP fit III
$\chi^2_{\min}/dof$ (quality of fit)	6.28/7 (51%)	7.92/7 (34%)	8.3/7 (31%)
$\gamma$	$71.7^\circ \pm 5.7^\circ$	$71.1^\circ \pm 8.4^\circ$	$53.2^\circ \pm 8.7^\circ$
$ T' $ (eV)	$5.21 \pm 0.27$	$5.23 \pm 0.28$	$5.40 \pm 0.30$
$\delta_{T'}$	$30.3^\circ \pm 5.5^\circ$	$32.9^\circ \pm 13.6^\circ$	$75.0^\circ \pm 40.9^\circ$
$ C' $ (eV)	$3.25 \pm 0.54$	$3.56 \pm 0.54$	$4.41 \pm 0.43$
$\delta_{C'}$	$-14.5^\circ \pm 18.7^\circ$	$-24.4^\circ \pm 15.4^\circ$	$-0.4^\circ \pm 35.8^\circ$
$ P' $ (eV)	$48.6 \pm 0.7$	$47.4 \pm 4.3$	$25.2 \pm 8.0$
$\delta_{NP}$	$7.6^\circ \pm 4.3^\circ$	$-5^\circ \pm 2^\circ$	$-100^\circ \pm 44^\circ$
$ P'_{EW,NP} $ (eV)	$20.1 \pm 4.7$	-	-
$\phi_{EW}$ (eV)	$-87.4^\circ \pm 4.5^\circ$	-	-
$ P'^C_{EW,NP} $ (eV)	-	$33.6 \pm 26.7$	-
$\phi^C_{EW}$ (eV)	-	$-88^\circ \pm 3^\circ$	-
$ P'_{NP} $ (eV)	-	-	$39.0 \pm 19.3$
$\phi_P$ (eV)	-	-	$-1.94^\circ \pm 2.12^\circ$

**Table 4:** Results for “NP fit I”, “NP fit II” and “NP fit III”. See the text for details.

Observable	NP fit I	NP fit II	NP fit III
$BR(B^+ \rightarrow \pi^+\pi^0)$	$3.5 \times 10^{-4}$	0.071	0.037
$BR(B^0 \rightarrow \pi^+\pi^-)$	0.056	0.086	0.030
$BR(B^0 \rightarrow \pi^0\pi^0)$	$2.4 \times 10^{-7}$	0.21	0.032
$BR(B^+ \rightarrow \pi^+K^0)$	1.7	0.13	0.089
$BR(B^+ \rightarrow \pi^0K^+)$	0.16	0.29	0.70
$BR(B^0 \rightarrow \pi^-K^+)$	1.1	0.026	0.97
$BR(B^0 \rightarrow \pi^0K^0)$	0.099	0.46	1.2
$A_{CP}(B^+ \rightarrow \pi^+\pi^0)$	0.028	0.028	0.028
$A_{CP}(B^0 \rightarrow \pi^+\pi^-)$	0.14 0	0.31	0.023
$A_{CP}(B^0 \rightarrow \pi^0\pi^0)$	0.021	0.037	0.48
$A_{CP}(B^+ \rightarrow \pi^+K^0)$	0.25	2.0	0.024
$A_{CP}(B^+ \rightarrow \pi^0K^+)$	0.13	0.42	0.15
$A_{CP}(B^0 \rightarrow \pi^-K^+)$	0.16	0.29	0.097
$A_{CP}(B^0 \rightarrow \pi^0K^0)$	1.8	0.0013	1.4
$S_{CP}(B^0 \rightarrow \pi^+\pi^-)$	0.18	0.058	0.042
$S_{CP}(B^0 \rightarrow \pi^0K^0)$	0.49	3.5	2.9

**Table 5:**  $\Delta\chi^2_{\min}$  — the contribution of each data point to the  $\chi^2_{\min}$ .

And therefore this assumption should be taken with care. Adopting the smallness of NP phases, we need to introduce just one NP amplitude with effective weak phase for each topological amplitude. We assume there is no NP contribution to tree amplitude  $T'$  and color-suppressed tree amplitude  $C'$ . Then the decay amplitudes can be written as

$$\sqrt{2}A(B^+ \rightarrow \pi^+\pi^0) = -(T + C)e^{i\gamma},$$

$$\begin{aligned}
A(B^0 \rightarrow \pi^+ \pi^-) &= -(T e^{i\gamma} + P e^{-i\beta}), \\
\sqrt{2}A(B^0 \rightarrow \pi^0 \pi^0) &= -C e^{i\gamma} + P e^{-i\beta}, \\
A(B^+ \rightarrow \pi^+ K^0) &= -P' + P'_{\text{NP}} e^{i\phi_P} - \frac{1}{3} P'_{\text{EW, NP}} e^{i\phi_{\text{EW}}^C}, \\
\sqrt{2}A(B^+ \rightarrow \pi^0 K^+) &= P' - T' e^{i\gamma} - C' e^{i\gamma} - P'_{\text{EW}} - P'_{\text{NP}} e^{i\phi_P} - P'_{\text{EW, NP}} e^{i\phi_{\text{EW}}} \\
&\quad - \frac{2}{3} P'_{\text{EW, NP}} e^{i\phi_{\text{EW}}^C}, \\
A(B^0 \rightarrow \pi^- K^+) &= P' - T' e^{i\gamma} - P'_{\text{NP}} e^{i\phi_P} - \frac{2}{3} P'_{\text{EW, NP}} e^{i\phi_{\text{EW}}^C}, \\
\sqrt{2}A(B^0 \rightarrow \pi^0 K^0) &= -P' - C' e^{i\gamma} - P'_{\text{EW}} + P'_{\text{NP}} e^{i\phi_P} - P'_{\text{EW, NP}} e^{i\phi_{\text{EW}}} \\
&\quad - \frac{1}{3} P'_{\text{EW, NP}} e^{i\phi_{\text{EW}}^C}.
\end{aligned} \tag{3.1}$$

Note that we included NP contribution to the color-suppressed electroweak diagram. This is because this contribution need not be suppressed compared to the electroweak penguin while it is actually suppressed in the SM.

This description of NP has 7 additional parameters, overall strong phase  $\delta_{\text{NP}}$  relative to that of  $P'$  which we set to be zero, three real NP amplitudes and three NP weak phases. Using all the new parameters in fitting makes statistics quite poor. So, at first, we assume one NP terms dominate and neglect the others.

First, we consider only the effect of  $P'_{\text{EW}}$  (**NP fit I**) which corresponds to the solution considered in [5]. Table 4 shows that we obtained an excellent fit for this scenario. In addition, we can see in table 5 that all the puzzling behaviors of the SM have disappeared. The largest deviation from the best fit parameters is at most  $1.3\sigma$ .

Now we consider a scenario where  $P'_{\text{EW, NP}}$  dominates (**NP fit II**). We can see from table 4 that this fit is also acceptable. However, the data for  $S_{\text{CP}}(B^0 \rightarrow \pi^0 K^0)$  is a little bit away from the best fitted values.

Similarly we can consider the case where only  $P'_{\text{NP}}$  exists (**NP fit III**). In this case, although  $\chi^2_{\text{min}}/\text{dof}$  is acceptable,  $\Delta\chi^2_{\text{min}}$  of  $S_{\text{CP}}(B^0 \rightarrow \pi^0 K^0)$  is not so satisfactory.

We can analyze more general cases of having two-types of NP simultaneously. We have 11 parameters to fit in these cases. For each case we obtained several acceptable solutions, which is due to the low statistics. With only  $P'_{\text{EW, NP}}$  and  $P'_{\text{EW, NP}} e^{i\phi_{\text{EW}}^C}$  (**NP fit IV**) we get two distinctive solutions in table 6. It is interesting to note that the best solution of “NP fit IV” favors the enhancement of both  $P'_{\text{EW, NP}}$  and  $P'_{\text{EW, NP}} e^{i\phi_{\text{EW}}^C}$  contrary to ref. [5]. As mentioned earlier,  $P'_{\text{EW, NP}} e^{i\phi_{\text{EW}}^C}$  is not necessarily color suppressed and can be as large as  $P'_{\text{EW, NP}}$ . The second solution corresponds to that found in ref. [5], i.e. NP in the electroweak penguin sector.

The scenario of having non-vanishing  $P'_{\text{NP}}$  and  $P'_{\text{EW, NP}}$  (**NP fit V**) shows that  $P'_{\text{NP}}$  need not be suppressed and can be almost as large as  $P'_{\text{EW, NP}}$ . Even when  $P'_{\text{EW, NP}}$  vanishes (**NP fit VI**), the large contribution of  $P'_{\text{NP}}$  and  $P'_{\text{EW, NP}} e^{i\phi_{\text{EW}}^C}$  can solve the  $B \rightarrow \pi K$  puzzle.

Now we consider the simultaneous contribution of all the possible NP contributions, i.e.  $P'_{\text{NP}}, P'_{\text{EW, NP}}$  and  $P'_{\text{EW, NP}} e^{i\phi_{\text{EW}}^C}$  for completeness (**NP fit VII**), although we have poor statistics for definite prediction. As expected, we obtained many physically acceptable local minima.

	NP fit IV	NP fit V	NP fit VI
$\chi^2_{\min}/dof$ (quality of fit)	2.45/5 (78.0%)	4.55/5 (47%)	0.51/5 (99%)
	2.72/5 (74.4%)		0.97/5 (97%)
$\gamma$	$69.7^\circ \pm 5.6^\circ$	$62.4^\circ \pm 8.4^\circ$	$60.7^\circ \pm 8.8^\circ$
	$75.5^\circ \pm 5.6^\circ$		$55.0^\circ \pm 7.1^\circ$
$ T' $ (eV)	$5.26 \pm 0.24$	$5.25 \pm 0.24$	$5.27 \pm 0.24$
	$5.29 \pm 0.28$		$5.35 \pm 0.27$
$\delta_{T'}$	$31.9^\circ \pm 9.0^\circ$	$43.1^\circ \pm 15.1^\circ$	$50.4^\circ \pm 23.3^\circ$
	$26.5^\circ \pm 7.3^\circ$		$70.5^\circ \pm 27.2^\circ$
$ C' $ (eV)	$3.71 \pm 0.50$	$4.03 \pm 0.58$	$4.15 \pm 0.51$
	$3.02 \pm 0.54$		$4.35 \pm 0.44$
$\delta_{C'}$	$-27.7^\circ \pm 11.1^\circ$	$-21.4^\circ \pm 16.5^\circ$	$-17.1^\circ \pm 18.9^\circ$
	$-13.8^\circ \pm 20.3^\circ$		$-2.36^\circ \pm 22.7^\circ$
$ P' $ (eV)	$40.0 \pm 2.6$	$32.6 \pm 11.9$	$30.6 \pm 11.0$
	$49.6 \pm 0.9$		$26.2 \pm 8.3$
$\delta_{NP}$	$177^\circ \pm 1^\circ$	$9.04^\circ \pm 6.95^\circ$	$-1.59^\circ \pm 4.60^\circ$
	$7.9^\circ \pm 4.8^\circ$		$-179^\circ \pm 5^\circ$
$P'_{EW,NP}$ (eV)	$62.2 \pm 6.9$	$21.1 \pm 4.7$	-
	$19.4 \pm 6.0$		-
$\phi'_{EW}$	$-75.9^\circ \pm 3.8^\circ$	$-87.2^\circ \pm 6.0^\circ$	-
	$-90.9^\circ \pm 5.0^\circ$		-
$P'^C_{EW,NP}$ (eV)	$65.4 \pm 7.4$	-	$86.4 \pm 19.5$
	$3.96 \pm 4.34$		$-61.7 \pm 21.1$
$\phi'^C_{EW}$	$106^\circ \pm 3^\circ$	-	$146^\circ \pm 23^\circ$
	$31.6^\circ \pm 78.3^\circ$		$27.7^\circ \pm 15.4^\circ$
$P'_{NP}$ (eV)	-	$16.9 \pm 12.2$	$56.7 \pm 13.3$
	-		$25.3 \pm 5.0$
$\phi'_P$	-	$-167^\circ \pm 16^\circ$	$6.91^\circ \pm 6.93^\circ$
	-		$-81.5^\circ \pm 23.6^\circ$

**Table 6:** The fits for NP (IV, V, VI). See the text for details.

We list just three of them in table 7. Since the  $\chi^2_{\min}$ 's of “NP fit VI” are quite low, the solutions with low-lying  $\chi^2_{\min}$  values look similar to those of “NP fit VI” (See the 1st solution). The 2nd solution shows that all the 3 types of NP can be sizable. The 3rd solution corresponds to the “NP fit I” which is the NP in the electroweak penguin sector obtained in [5].

#### 4. Conclusions

We performed  $\chi^2$  fitting to check if the conventional parametrization in the SM describes well the current experimental data of  $B \rightarrow \pi\pi$  and  $B \rightarrow \pi K$  decays. Contrary to the data

NP fit VII						
$\chi_{\min}^2/dof$	$\gamma$	$ T' $	$\delta_{T'}$	$ C' $	$\delta_{C'}$	$ P' $
0.21(98%)	$63 \pm 10$	$5.3 \pm 0.2$	$44 \pm 23$	$4.1 \pm 0.6$	$-23 \pm 19$	$33 \pm 12$
0.43(93%)	$73 \pm 8$	$5.2 \pm 0.3$	$73 \pm 8$	$3.3 \pm 0.8$	$-15 \pm 21$	$46 \pm 12$
2.6 (45%)	$69 \pm 31$	$5.2 \pm 0.3$	$33 \pm 41$	$3.6 \pm 2.5$	$-20 \pm 25$	$41 \pm 41$
$\delta_{\text{NP}}$	$P'_{\text{EW,NP}}$	$\phi'_{\text{EW}}$	$P'^C_{\text{EW,NP}}$	$\phi'^C_{\text{EW}}$	$P'_{\text{NP}}$	$\phi'_P$
$179 \pm 2$	$2.1 \pm 3.8$	$9.8 \pm 52$	$76 \pm 21$	$-93 \pm 22$	$15 \pm 8$	$104 \pm 37$
$0.6 \pm 1.3$	$65 \pm 45$	$33 \pm 29$	$65 \pm 44$	$-140 \pm 30$	$48 \pm 24$	$48 \pm 38$
$-172 \pm 8$	$19 \pm 9$	$89 \pm 7$	$2.7 \pm 3.9$	$-180 \pm 340$	$8.6 \pm 42$	$5.7 \pm 38$

**Table 7:** The fit for “NP fit VII”.

used in ref. [10], the current data disfavors this parametrization given in (2.5) and (2.12) at 2–3  $\sigma$  level.

We interpreted this difficulty in the SM as a manifestation of NP and investigated various NP solutions. When a single NP amplitude dominates, NP in the electroweak penguin sector is the most favorable solution in accord with [5]. When two or more NP amplitudes exist simultaneously, solutions other than in the electroweak penguin sector can also explain the deviation very well.

## Acknowledgments

The author thanks C.S. Kim for useful comments and P. Ko for warm hospitality during his visit to KIAS where part of this work was done.

## References

- [1] M. Gronau and D. London, *Isospin analysis of CP asymmetries in B decays*, *Phys. Rev. Lett.* **65** (1990) 3381.
- [2] Y. Nir and H.R. Quinn, *Measuring ckm parameters with CP asymmetry and isospin analysis in  $B \rightarrow \pi K$* , *Phys. Rev. Lett.* **67** (1991) 541;  
 M. Gronau, J.L. Rosner and D. London, *Weak coupling phase from decays of charged B mesons to  $\pi K$  and  $\pi\pi$* , *Phys. Rev. Lett.* **73** (1994) 21 [[hep-ph/9404282](#)];  
 R. Fleischer, *Strategies for fixing the CKM-angle  $\gamma$  and obtaining experimental insights into the world of electroweak penguins*, *Phys. Lett.* **B 365** (1996) 399 [[hep-ph/9509204](#)];  
 Y.Y. Keum, H.-N. Li and A.I. Sanda, *Penguin enhancement and  $B \rightarrow K\pi$  decays in perturbative QCD*, *Phys. Rev.* **D 63** (2001) 054008 [[hep-ph/0004173](#)];  
 M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, *QCD factorization in  $B \rightarrow \pi K, \pi\pi$  decays and extraction of Wolfenstein parameters*, *Nucl. Phys.* **B 606** (2001) 245 [[hep-ph/0104110](#)].
- [3] A.J. Buras and R. Fleischer, *Constraints on the CKM angle  $\gamma$  and strong phases from  $B \rightarrow \pi K$  decays*, *Eur. Phys. J.* **C 16** (2000) 97 [[hep-ph/0003323](#)].
- [4] M. Imbeault, A. S.-L. Lemerle, V. Page and D. London, *Obtaining the full unitarity triangle from  $B \rightarrow \pi K$  decays*, *Phys. Rev. Lett.* **92** (2004) 081801 [[hep-ph/0309061](#)].

- [5] A.J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, *The  $B \rightarrow \pi K$  puzzle and its relation to rare  $B$  and  $K$  decays*, *Eur. Phys. J. C* **32** (2003) 45 [[hep-ph/0309012](#)];  
 A.J. Buras, R. Fleischer, S. Recksiegel and F. Schwab,  *$B \rightarrow \pi\pi$ , new physics in  $B \rightarrow \pi K$  and implications for rare  $K$  and  $B$  decays*, *Phys. Rev. Lett.* **92** (2004) 101804 [[hep-ph/0312259](#)];  
 A.J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, *Anatomy of prominent  $B$  and  $K$  decays and signatures of CP-violating new physics in the electroweak penguin sector*, *Nucl. Phys. B* **697** (2004) 133 [[hep-ph/0402112](#)];  
 A.J. Buras, R. Fleischer, S. Recksiegel and F. Schwab, *New aspects of  $B \rightarrow \pi\pi, \pi K$  and their implications for rare decays*, *Eur. Phys. J. C* **45** (2006) 701 [[hep-ph/0512032](#)].
- [6] S. Baek, P. Hamel, D. London, A. Datta and D.A. Suprun, *The  $B \rightarrow \pi K$  puzzle and new physics*, *Phys. Rev. D* **71** (2005) 057502 [[hep-ph/0412086](#)].
- [7] A. Datta and D. London, *Measuring new-physics parameters in  $B$  penguin decays*, *Phys. Lett. B* **595** (2004) 453 [[hep-ph/0404130](#)].
- [8] S. Baek, D. London, J. Matias and J. Virto, *Supersymmetric contributions to  $B_s^0 \rightarrow K^+ K^-$* , *JHEP* **02** (2006) 027 [[hep-ph/0511295](#)].
- [9] S. Baek, F.J. Botella, D. London and J.P. Silva, *Measurements of new physics in  $B \rightarrow \pi\pi$  decays*, *Phys. Rev. D* **72** (2005) 114007 [[hep-ph/0509322](#)];  
 S. Baek, F.J. Botella, D. London and J.P. Silva, *Can one detect new physics in  $i = 0$  and/or  $i = 2$  contributions to the decays  $B \rightarrow \pi\pi$ ?*, *Phys. Rev. D* **72** (2005) 036004 [[hep-ph/0506075](#)].
- [10] C.-W. Chiang, M. Gronau, J.L. Rosner and D.A. Suprun, *Charmless  $B \rightarrow pp$  decays using flavor SU(3) symmetry*, *Phys. Rev. D* **70** (2004) 034020 [[hep-ph/0404073](#)].
- [11] M. Gronau, O.F. Hernandez, D. London and J.L. Rosner, *Decays of  $B$  mesons to two light pseudoscalars*, *Phys. Rev. D* **50** (1994) 4529 [[hep-ph/9404283](#)].
- [12] See, for example, G. Buchalla, A.J. Buras and M.E. Lautenbacher, *Weak decays beyond leading logarithms*, *Rev. Mod. Phys.* **68** (1996) 1125 [[hep-ph/9512380](#)].
- [13] M. Gronau, D. Pirjol and T.-M. Yan, *Model-independent electroweak penguins in  $B$  decays to two pseudoscalars*, *Phys. Rev. D* **60** (1999) 034021 [[hep-ph/9810482](#)], erratum *ibid.* **69** (2004) 119901.
- [14] Heavy Flavor Averaging Group, <http://www.slac.stanford.edu/xorg/hfag/>.
- [15] H.-N. Li and S. Mishima, *Implication of the  $B \rightarrow \rho\rho$  data on the  $B \rightarrow \pi\pi$  puzzle*, *Phys. Rev. D* **73** (2006) 114014 [[hep-ph/0602214](#)].
- [16] Y. Grossman, M. Neubert and A.L. Kagan, *Trojan penguins and isospin violation in hadronic  $B$  decays*, *JHEP* **10** (1999) 029 [[hep-ph/9909297](#)];  
 T. Yoshikawa, *A possibility of large electro-weak penguin contribution in  $B \rightarrow K\pi$  modes*, *Phys. Rev. D* **68** (2003) 054023 [[hep-ph/0306147](#)];  
 S. Nandi and A. Kundu, *Large electroweak penguins in  $B \rightarrow \pi\pi$  and  $B \rightarrow \pi K$ : implication for new physics*, [hep-ph/0407061](#);  
 S. Mishima and T. Yoshikawa, *Large electroweak penguin contribution in  $B \rightarrow K\pi$  and  $\pi\pi$  decay modes*, *Phys. Rev. D* **70** (2004) 094024 [[hep-ph/0408090](#)];  
 T. Morozumi, Z.H. Xiong and T. Yoshikawa,  *$B \rightarrow X_s \ell^+ \ell^-$  and  $B \rightarrow K\pi$  decays in vectorlike quark model*, [hep-ph/0408297](#);  
 Y.-Y. Charng and H.-n. Li, *Weak phases from the  $B \rightarrow \pi\pi, K\pi$  decays*, *Phys. Rev. D* **71** (2005) 014036 [[hep-ph/0410005](#)];

- X.-G. He and B.H.J. McKellar, *Hadron decay amplitudes from  $B \rightarrow K\pi$  and  $B \rightarrow \pi\pi$  decays*, hep-ph/0410098;
- T. Carruthers and B.H.J. McKellar, *Implications of  $B$  meson decay data: a graphical approach*, hep-ph/0412202;
- Y.L. Wu and Y.F. Zhou, *Charmless decays  $B \rightarrow \pi\pi, \pi K$  and  $KK$  in broken  $SU(3)$  symmetry*, *Phys. Rev. D* **72** (2005) 034037 [hep-ph/0503077];
- M. Gronau and J.L. Rosner, *The  $B \rightarrow S$  penguin amplitude in charmless  $B \rightarrow pp$  decays*, *Phys. Rev. D* **71** (2005) 074019 [hep-ph/0503131];
- C.S. Kim, S. Oh and C. Yu, *A critical study of the  $B \rightarrow K\pi$  puzzle*, *Phys. Rev. D* **72** (2005) 074005 [hep-ph/0505060];
- S. Khalil, *CP asymmetries and branching ratios of  $B \rightarrow K\pi$  in supersymmetric models*, *Phys. Rev. D* **72** (2005) 035007 [hep-ph/0505151];
- X.-Q. Li and Y.-D. Yang, *Revisiting  $B \rightarrow \pi\pi, \pi K$  decays in QCD factorization approach*, *Phys. Rev. D* **72** (2005) 074007 [hep-ph/0508079];
- H.N. Li, S. Mishima and A.I. Sanda, *Resolution to the  $B \rightarrow \pi K$  puzzle*, *Phys. Rev. D* **72** (2005) 114005 [hep-ph/0508041];
- R. Arnowitt, B. Dutta, B. Hu and S. Oh, *The  $B \rightarrow K\pi$  puzzle and supersymmetric models*, *Phys. Lett. B* **633** (2006) 748 [hep-ph/0509233];
- C.W. Bauer, I.Z. Rothstein and I.W. Stewart, *Scet analysis of  $B \rightarrow K\pi, B \rightarrow K\bar{K}$  and  $B \rightarrow \pi\pi$  decays*, hep-ph/0510241;
- W.-S. Hou, M. Nagashima, G. Raz and A. Soddu, *Four generation CP-violation in  $B \rightarrow \phi K^0, \pi^0 K^0, \eta' K^0$  and hadronic uncertainties*, hep-ph/0603097.
- [17] See, for example, CKMfitter, <http://ckmfitter.in2p3.fr/>.